STRESS ANALYSIS OF CONCRETE STRUCTURES SUBJECTED TO VARIABLE THERMAL LOADS

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ABSTRACT: Predicting stress levels caused by thermal effects is a main factor in the correct design of structures; both relate to aspects of the maximum stress limitation and the durability of constructions. A numerical procedure based on the finite-element method for the stress-strain analysis of concrete structures exposed to time- and space-variable thermal loads is presented in this paper. Different environmental conditions can be simulated by means of suitable boundary conditions imposed on the differential equations governing the phenomenon. In particular, the seasonal and daily variation of external temperature is considered. The presented procedure can also consider the heat generation phenomenon, due to the hydration reaction of the cement, which can cause considerable temperature gradient and related mechanical stresses, especially for thick concrete casting. The stress-strain-thermal analyses of a concrete dam (Sistri dam, Sardegna, Italy) and a typical bridge section (i.e., box-girder cross section) are carried out to prove the effectiveness and reliability of this numerical method in practical structural design.

INTRODUCTION

Thermal effects have frequently been related to the damage to concrete bridges and dams. Therefore, the prediction of stress levels due to time-variable and space-variable thermal loads is of fundamental relevance in the correct design of structures considering the aspects of the maximum stress limitation and the durability of constructions. This question is particularly important in concrete structures because of the considerable heat generated in the hydration reaction, the low heat conductivity of the material (e.g., in concrete gravity dams), and the variable temperature distribution that may arise (e.g., in concrete bridges). Interaction with the external temperature and the solar radiation leads to seasonal and daily temperature gradient in the structure [e.g., Elbadry and Ghali (1983), “Thermal effects in concrete structures” (1985), “Thermal effects” (1982)]. As a result, a great temperature gradient and a related no-negligible stress gradient are induced in such structures. However, the intrinsic difficulty of thermal analysis and the inability to model an effective environmental condition often lead the designer to consider the effects of thermal loads in a simplified way, which may lead to unreliable results.

A finite-element procedure for the stress-strain analysis in concrete structures exposed to time-variable environmental conditions is presented in this paper. This technique takes into account the heat generation phenomenon in massive concrete structures and the seasonal and daily variation of temperature, allowing different structure configurations and thermal conditions to be studied. Both the temperature profiles and stress states due to temperature changes, during and after the construction phases, can be evaluated. These characteristics make the proposed method very useful and innovative in the field of the structural design of concrete structures for which thermal effects have to be considered. Moreover, the possibility to perform a full three-dimensional analysis and update the mesh during the analysis allows the study of the evolution of the construction of monolithic structures (such as gravity dams) in detail.

The analyses of a concrete gravity dam and a typical bridge section (i.e., a box-girder cross section) are carried out to prove the effectiveness and reliability of this numerical method.

HEAT CONDUCTION

Governing Equations

The heat conduction is governed by the well-known Fourier law

$$\rho \cdot C_p \frac{\partial T}{\partial t} = \lambda \cdot \text{div}(T) + H$$

where $T$ = temperature, function of the point coordinates $x$, and the time $t$; $C_p$ = specific heat capacity of concrete; $\rho$ = density of the material; $\lambda$ = isotropic thermal conductivity of concrete.
which is assumed to be independent of direction; and $H =$ time rate of heat generated per unit volume. For example, $H$ can be the heat of the hydration reaction of cement.

It is worthwhile to prove that the parameters $C_p$, $\lambda$, and $\rho$ in (1) depend on the temperature, the stress and strain level at the specific point, and the rate of cement hydration in concrete. If all these effects are taken into account, the equation of heat conduction becomes nonlinear and its analytical solution is very difficult to obtain. Therefore, a numerical method should be used, especially in the analysis of complex structures.

Depending on the problem to be studied, some simplified hypotheses can be made without introducing too many restrictive limitations. For example, in the definition of the environmental conditions, which are variable with seasonal and daily cycles, the temperature can be assumed to range between 0°C and 50°C. In this range, if only aged concrete is considered, the nonlinear effects of temperature distribution can be neglected. Moreover, in the thermal analysis of aged concrete the term $H$ in (1) can be set equal to zero.

**Initial and Boundary Conditions in Thermal Analysis**

The initial conditions associated with (1) can be defined by means of a function of coordinates $x_i$

$$T(x_i, t = 0) = T_0(x_i) \quad \text{at } t = 0$$

while the boundary conditions may be described in two different ways.

**Surface with Imposed Temperature**

$$T(x_i, t) = f(x_i, t), \quad x_i \in \Sigma$$

where $\Sigma =$ boundary of the investigated domain; and $f(x_i, t)$ = a known function of coordinates $x_i$ and time $t$.

**Surface with Convective and Radiant Thermal Exchanges**

The thermal flux through an unitary area can be written as [e.g., Froli (1993)]

$$q = \lambda \frac{\partial T}{\partial n} = q_c + q_n + q_{ew} + q_{em} \quad \text{on } x_i \in \Sigma$$

where $n =$ unit vector normal to the surface; $q_c =$ convective heat exchange; $q_n =$ thermal power due to the direct and diffuse solar radiation; $q_{ew}$ and $q_{em} =$ thermal power produced by the atmosphere and the earth, respectively, and absorbed through the material surface; and $q_{er} =$ emissivity of the surface.

The convective heat exchange $q_c$ occurs between the surface $\Sigma$ and the surrounding fluid, and depends on the temperature difference between the air ($T_a$) and the structure surface ($T$). The phenomenon can be approximately expressed by Newton’s convection law [e.g., Branco and Mendes (1993)]

$$q_c = \alpha_c \cdot (T_a - T) \quad \text{on } x_i \in \Sigma$$

where the convective coefficient $\alpha_c$ depends on the intrinsic characteristics of the fluid and the surface, and on the fluid velocity distribution (i.e., it varies according to the mechanical characteristics of fluid, the problem geometry, and the temperature distribution itself).

In a forced ventilation condition (e.g., the open air environment) $\alpha_c$ is strongly influenced by the external fluid velocity $V$, since the amount of heat carried away from the surface by convective effect is directly proportional to the external fluid velocity itself.

Fig. 1 shows the temperature profiles obtained from experimental tests in some concrete slabs, for different values of the wind speed ("Thermal" 1982). In the simulation of natural convective exchange involving air-concrete interface, the following relationships between $\alpha_c$ and $V$ can be used [e.g., Froli (1993)]

$$\alpha_c = 5.6 + 4.0 \cdot V \quad \text{for } V < 5 \text{ m/sec}; \quad \alpha_c = 7.15 \cdot V^{0.78} \quad \text{for } V \geq 5 \text{ m/sec}$$

The thermal power $q_n$, caused by the direct and diffuse solar radiation, can be written as

$$q_n = a \cdot (i + d) \quad \text{on } x_i \in \Sigma$$

where $a =$ absorption coefficient of surface; $i =$ direct solar radiation power reaching the surface; and $d =$ diffuse solar radiation power reaching the surface.

The $i$ and $d$ values are complex functions of the environmental variables (i.e., the geographical latitude, solar constant, air opacity, solar azimuth, and so on); $i$ also depends on the relative position of the surface to the sun’s position (e.g., $i = 0$ for a shaded surface). Since the values
of $i$ and $d$ depend on many factors, their exact estimation must involve experimental methods, both in situ and in the laboratory.

In (4) the sum $q_v + q_w + q_a$ can be neglected when compared with the other terms $q_v + q_a$. This simplification leads to a more convenient form of (4), in which the whole superficial thermal transmission can be expressed as an equivalent convective exchange between the body's surface at temperature $T$ and the external fluid at a suitable fictitious temperature $T^\circ_f$

$$\lambda \frac{dT}{dn} = \alpha_c \cdot (T^\circ_f - T)$$

with

$$T^\circ_f = T_c + \frac{d}{\alpha_c} (i + d)$$

The fictitious temperature $T^\circ_f$ is termed "equivalent air temperature" because it includes both the effect of air temperature and heat from the sun.

LINEAR ELASTIC STRESS-STRAIN RELATIONSHIPS INCLUDING THERMAL EFFECTS

To evaluate the stress-strain state and the displacement field due to any distribution of thermal loads, some hypotheses can be assumed [e.g., Zienkiewicz and Taylor (1991)]

- Uncoupled temperature and stress fields
- Infinitesimal strains and displacements
- Linear elastic behavior of material

The first statement allows solving the problems of heat conduction and stress analysis independently. At a fixed time $t$ the thermal transient is solved; then, once the temperature field is known, the stress-strain state can be estimated.

From the second assumption, the usual linear relationships for small strain can be written [e.g., Zienkiewicz and Taylor (1991)]

$$\varepsilon_{ij} = \frac{1}{2} \cdot u_{ij}$$

where $\varepsilon_{ij}$ = total strain tensor; and $u_{ij}$ = derivative of the displacement field.

The linear elastic behavior of material can be described by the well-known generalized Hooke's law [e.g., Zienkiewicz and Taylor (1991)]

$$\sigma_{ij} = D_{ijkl} \cdot (\varepsilon_{kl} - \varepsilon_{r,kl} - \varepsilon_{0,kl}) + \sigma_{ij}^0$$

where $\sigma_{ij}$ = Cauchy stress tensor; $D_{ijkl}$ = tensor of elastic constant; $\varepsilon_{r,kl}$ = strain due to thermal expansion; $\varepsilon_{0,kl}$ = initial strain; and $\sigma_{ij}^0$ = initial stress corresponding to $\varepsilon_{0,kl}$.

Eq. (12) states that at every point and at every time, the stress field depends only on the initial stress $\sigma_{ij}^0$ and on the elastic component of total strain ($\varepsilon_{kl} - \varepsilon_{r,kl} - \varepsilon_{0,kl}$).
The thermal strain \( \varepsilon_{T,\lambda} \) due to the thermal expansion, in an isotropic material is given by [e.g., Lewis (1981)]

\[
\varepsilon_{T,\lambda} = \delta_{\lambda k} \beta (T - T_0)
\]  

(13)

where \( \delta_{\lambda k} = \text{Kronecker delta} \) (i.e., \( \delta_{\lambda k} = 1 \) if \( k = 1 \) and \( \delta_{\lambda k} = 0 \) if \( k \neq 1 \)); \( \beta \) = coefficient of thermal expansion; \( T_0 \) = temperature in correspondence with which the thermal strain level is zero; and \( \Delta T = (T - T_0) \) = temperature variation.

**Boundary Condition in Stress-Strain Analysis**

The boundary \( \Sigma \) of the domain can be divided into the free portion \( \Sigma_f \) and the bound portion \( \Sigma_b \) so that the boundary condition for stress and displacements fields can be written as

\[
\sigma_{\lambda \mu} \cdot n_{\lambda} = t_{\lambda} \quad \text{on} \quad \Sigma_f
\]  

(14)

and

\[
u_{\lambda} = U_{\lambda} \quad \text{on} \quad \Sigma_b
\]  

(15)

where \( n_{\lambda} \) = unit vector normal to the boundary surface \( \Sigma \); \( t_{\lambda} \) = external load applied on the surface \( \Sigma_f \); and \( U_{\lambda} \) = prescribed displacements on surface \( \Sigma_b \).

**FINITE-ELEMENT FORMULATION**

Analytical solutions to the equations governing the phenomenon of heat conduction and the stress-strain related problem [i.e., (1), (11), and (12)] are very difficult to obtain, especially for great and geometrically complex structures, and a numerical approach should be preferred. The finite-element procedure for spatial discretization and the generalized trapezoidal methods for time integration are used in the present formulation.

Only the numerical formulation of the heat conduction problem is presented in this section, since the finite-element analysis of linear elastic continua is well known, and fully and extensively discussed in many publications [e.g., Zienkiewicz and Taylor (1991)].

Within the classical finite-element method, temperature values at the nodes of the finite-element mesh are assumed to be basic variables and, therefore, the temperature field can be expressed as

\[
T(x_i, t) = N_j(x_i) \cdot T_j(t)
\]  

(16)

where \( N_j \) = shape function of the generic finite element at node \( j \); and \( T_j(t) \) = temperature value at the same node \( j \) at time \( t \).

After spatial discretization, the heat transfer (1) assumes the following form:

\[
C_T \cdot \frac{d}{dt} T_j(t) + K_{\lambda \mu} \cdot T_j(t) = Q_j(t)
\]  

(17)

where \( C_T \) and \( K_{\lambda \mu} \) = matrices of thermal capacity and heat conductivity of the system; \( T_j(t) \) = time derivative of the temperature vector; and \( Q_j \) = vector of nodal heat flux evaluated at time \( t \). The expressions of the matrices \( C_T \) and \( K_{\lambda \mu} \) are

\[
C_T = \int \nabla N_j \cdot \nabla N_i \cdot \rho \cdot C_v \cdot dV; \quad K_{\lambda \mu} = \int \nabla N_j \cdot \lambda_{\lambda \mu} \cdot N_i \cdot dV
\]  

(18, 19)

where \( N_j \) = derivative of the shape functions; and \( \lambda_{\lambda \mu} \) = matrix of the material conductivity coefficients along the coordinate direction, which becomes a diagonal matrix for an isotropic material.

The matrix \( C_T \) is symmetric and definite positive. It is worth noting that to reduce computational efforts such matrix could be lumped by condensation [e.g., Zienkiewicz and Taylor (1991)].

The vector \( Q_j \) depends on the internal heat generation and the boundary conditions. It can be written as the sum of two components

\[
Q_j = Q_{j,1} + Q_{j,2}
\]  

(20)

The term \( Q_{j,1} \), termed equivalent convective flux at time \( t \), can be obtained from the imposition of the boundary condition through (9)

\[
Q_{j,1} = \int \nabla N_j \cdot \alpha \cdot (T_\ast - T_j) \cdot d\Sigma = \int \nabla N_j \cdot \alpha \cdot T_\ast \cdot d\Sigma - \int \alpha \cdot N_j \cdot T_j \cdot d\Sigma
\]  

(21)

the first term \( f_\Sigma N_j \cdot \alpha \cdot T_\ast \cdot d\Sigma \) = vector of equivalent nodal loads, and the second one can be written as

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\[
\left( \int_\Sigma \alpha_i \cdot N_i \cdot N_j \cdot d\Sigma \right) \cdot T_j = K_{ii}^* \cdot T_i \tag{22}
\]

and can be assembled with the stiffness matrix \( K_{ii} \) in (17)

\[
C_i \cdot T_i (t) + (K_{ii} + K_{ii}^*) \cdot T_i (t) = Q_i (t) \tag{23}
\]

The internal heat component flux \( Q_n^{in} \) is given by

\[
Q_n^{in} = \int_V N_i \cdot \rho \cdot H(T) \cdot dV \tag{24}
\]

where \( H(t) \) is known function of the velocity of heat generation into a unit volume.

The semidiscrete (17) can be integrated in time by using one algorithm of the generalized trapezoidal family of methods, which leads to [e.g., Ziennicki and Taylor (1991)]

\[
(C + \omega \cdot \Delta t \cdot K) T^{n+1} = (C + (1 - \omega) \cdot \Delta t \cdot K) T^n + \Delta t \cdot (\omega \cdot Q_n^{in} + (1 - \omega) \cdot Q^n) \tag{25}
\]

where \( \Delta t = t_{n+1} - t_n \) \((n = 1, 2, \ldots)\) denotes the time step and the apex \( n \) means that the term is evaluated at time \( t_n \). As is already known, for \( 0.5 \leq \omega < 1 \) the algorithm is unconditionally stable, and for \( \omega = 0.5 \) the order of accuracy is 2 [e.g., Ziennicki and Taylor (1991)].

**VALIDATION AND NUMERICAL APPLICATION**

The aforementioned relationships have been implemented in a finite-element algorithm developed to investigate two- and three-dimensional heat transient in a homogeneous isotropic material. The finite-element code performs the stress-strain analysis of a plain or axial-symmetric two-dimensional body, both in stationary or dynamic thermal conditions, as well as full three-dimensional stress analysis. Time variability of the mesh geometry can be considered in order to simulate successive phases of the structure’s construction.

**Sa Stria Dam Analysis**

One of the most effective applications of the proposed procedure is the analysis of the thick concrete casting (e.g., concrete gravity dams), in which the great amount of hydration heat generated, coupled with the material’s low heat diffusivity, causes considerable temperature gradients and mechanical stresses.

A conclusive analysis on the numerical prediction of the stresses that were introduced into the Sa Stria dam in Sardinia during construction phases and the first months of utilization is carried out in this paper. Extensive information on dam construction methods and on the results of a numerical simulation obtained by means of a monodimensional finite-difference integration compared with a two-dimensional finite-element procedure (i.e., the previous version of the present formulation) can be found in Fassò et al. (1991).

The Sa Stria dam, built in Sardegna (Italy), is a typical roller compacted concrete dam (RCC) with a maximum height of 87 m and a crest length of 350 m. The expansion joints, starting from 40 m high up to the top of the crest, are placed every 25 m. The dam (Fig. 2) consists of upstream and downstream facing elements, made of a conventional concrete containing 220 kg/m³ of class 225 low-heat pozzolanic cement, and a RCC core element, made of concrete containing 105 kg/m³ of class 325 pozzolanic cement and 105 kg/m³ of fly ash. The concrete was cast in 30-cm-thick horizontal lifts, each layer being placed in 2 days. This placement method induces a greater dispersion of heat (therefore, a reduced stress level) because of the high surface-to-volume ratio during the first 2 days after casting.

Covering a given lift with successive lifts on top gives rise to an increasing insulation in the vertical direction, while thermal exchange by convection continues undisturbed along the two dam faces; this leads to a rapid cooling of the cortical thickness and a virtually adiabatic increase in the core temperature.

**Numerical Results**

To reduce the computational efforts, 5-m-thick concrete lifts, each one cast every 20 days, were assumed in numerical simulation. A preliminary investigation validated this choice equivalent to real conditions from the standpoint of thermal analysis.

The numerical simulation of the phenomenon covered a 1,240-day-long period starting from the casting of the first layer. The following values were assumed for physical quantities: \( \lambda = 1.3 \text{ W/m}\cdot\text{°C} \) (for bedrock foundation); \( \lambda = 1.81 \text{ W/m}\cdot\text{°C} \) (for both concrete types); \( \alpha = 25.8 \text{ W/m}^2\cdot\text{°C} \); \( C_p = 0.22 \text{ J/kg}\cdot\text{°C} \); \( \gamma = 2,350 \text{ kg/m}^3 \) (for bedrock foundation); \( \gamma = 2,350 \text{ kg/m}^3 \) (for RCC concrete); \( \gamma = 2,450 \text{ kg/m}^3 \) (for standard concrete); and \( \beta = 10^{-5} \text{ °C}^{-1} \).

The law approximating the heat hydration progress is
\[ H(t) = P \cdot g(t) = P \cdot \left( \frac{1}{864} \cdot \exp(3.47 + 0.4 \cdot \ln(t - t_{\text{in}})) - 0.475 \cdot \ln(t - t_{\text{in}}) \right) \]  (26)

where \( P \) (kg/m\(^3\)) = cement content; and \( g(t) \) (W/m\(^3\)) is derived by experimental measurement.

The trend of the averaged daily air temperature \( (T^*_d) \) used in transient analysis is plotted in Fig. 3 for the upstream and downstream faces, respectively. About 8 months after the end of construction the upstream face comes in contact with the storage water that has a constant temperature of 6°C.

Figs. 4–7 show the characteristic isotherms obtained into the dam mass after 320, 560, 680, and 1,120 days of simulation, respectively.

During the initial phases, until the water supply, the thermal distribution into the mass clearly shows two distinct inner zones of higher temperature. The first zone is in the upper part of the dam and is due to the higher cement content of the concrete used; the second zone appears in the lower part of dam, where the greater material thickness is responsible for a hard heat dispersion.

As time passes (Figs. 6 and 7), the temperature tends to conform in the inner zone; near the
dam surface, however, considerable gradients remain. Further, temperature gradients greatly increase in the internal face, when it comes in contact with the external water (Fig. 6).

As soon as the temperature distribution $T$ into the concrete mass is known at every time $t$, the resulting tension $\sigma_x$ within the concrete can be easily obtained through the hypothesis of the material's linear elastic behavior.

To completely define the initial conditions, it is necessary to assume an "initial reference temperature" in correspondence with which the stress level is zero.

Any arbitrary definition of an initial reference temperature for the dam analysis influences the modeling of the stress-strain concrete behavior. For this reason some tests have been made to choose a suitable value for the reference temperature by comparing the numerical results with the experimental value measured in the Sa Stria dam, (Fassò et al. 1990). The temperature that appears repeatedly in each lift about 30 days after concrete casting has been assumed to be the initial temperature value, mainly for the following two reasons:

1. About 30 days after concrete casting the temperature becomes quasistationary in each lift and remains almost unchanged until water is supplied.
2. In the first month after concrete casting, the stress state that arises in each lift is very small because of the reduced stiffness of concrete during the early period, and is considerably lessened by creep effect.

However, a more accurate analysis might consider the additional effects of creep and changes in the mechanical characteristics of concrete with ageing, in detail.

The elastic characteristics of materials used in stress analysis are $E = 16 \text{ MPa}$ (Young modulus) and $v = 0.25$ (Poisson's ratio) for the bedrock foundation; $E = 28 \text{ MPa}$ and $v = 0.15$ for the conventional concrete; and $E = 25 \text{ MPa}$ and $v = 0.17$ for the RCC concrete.

The stress distribution is computed with a full three-dimensional stress analysis of a 25-m-long dam portion. The transversal stress distribution at time $t = 680$ days is shown in Fig. 8.
FIG. 8. Transversal Stress Distribution (MPa) at Time $t = 680$ Days for Three-Dimensional Analysis of Sa Stria Dam

The stress analysis shows that in the dam skin, which is at a lower temperature, tensile stresses appear while the core undergoes compressive stresses. The maximum tensile stress appears at time $t = 680$ days, in correspondence with the upper internal surface. Its value is about $0.5 \div 0.6$ MPa. The greatness of the tensional state depends on the reciprocal distance between shrinkage joints.

The longitudinal stresses are neutralized because of the presence of shrinkage joints. Actually, the same analysis carried out without the joints shows the longitudinal tensile stresses overcome the maximum admissible traction.

Box-Girder Section Analysis

Great effort has been put into the evaluation of thermal effects for concrete bridges [e.g., Mirambell and Aguado (1988), Branco and Mendes (1993), Priestley (1972, 1978), Zichner (1982)] since it is fundamental to the correct design of the structure.

In this context, one of the most exciting applications of the numerical code presented in this paper is the prediction of the mechanical stress induced by thermal actions in the box-girder sections, in which great thermal gradients, and consequently considerable stress gradient, can be easily originated because of the high specific surface (i.e., the surface-to-volume ratio).

In the following section, numerical results obtained in the modeling of the natural exposure conditions of a box-girder section situated in northern Italy's climatic zone are presented. Comparisons are made with experimental results reported in the literature.

The exposure boundary conditions are imposed by using (9) and by considering the $T^*_s$ formulations suggested in the CEB Bulletin ("Thermal" 1985). As an example, the following formulas can be used for surfaces that form a right angle at the horizontal plane with the direction of the sun's rays:

$$T^*_s = T_e + \frac{(a \cdot r_s + e_s \cdot \sigma \cdot T^*_s - e_s \cdot \sigma \cdot T^*_s)}{\alpha_s} \quad \text{upper horizontal plane}$$

$$T^*_s = T_e + \frac{0.25 \cdot a \cdot r_s + e_s \cdot \sigma \cdot (T^*_s - T^*_s \cdot e_s)}{\alpha_s} \quad \text{lower horizontal plane}$$

$$T^*_s = T_e + \frac{(a \cdot r_s + e_s \cdot \sigma \cdot T^*_s - e_s \cdot \sigma \cdot T^*_s)}{\alpha_s} \quad \text{vertical side}$$

$$T^*_s = T_e \quad \text{inner zones}$$

where the coefficients assume the following values in concrete sections asphalted on the upper face:

$$r_s = i + d \quad \text{global solar radiation}$$

$$e_s = 0.82 \quad \text{for clear atmosphere}$$

$$e_s = 0.99 \quad \text{hearth emissivity}$$

$$a = 0.87 \quad \text{absorption coefficient of asphalt}$$

$$a = 0.65 \quad \text{absorption coefficient of concrete}$$
\[ \varepsilon_i = 0.6 \quad \text{emissivity coefficient of asphalt} \]  

(36)

\[ \varepsilon_i = 0.88 \quad \text{emissivity coefficient of concrete} \]  

(37)

and \( \sigma = 10^{-8} \text{ W/m/K} \) is the Stefan-Boltzmann constant.

These relationships, coupled with the \( T_e \) and \( \alpha_e \), daily progress drawn in Fig. 9 (typical of the investigated geographical zone) give the graph in Fig. 10, which is used in the present investigation. The temperature \( T \) in (27)–(30) is averaged on the structure’s surface.

**FIG. 9.** \( \alpha_T \)-Time and Air Temperature–Time Relationships, Characteristic of North Italy’s Climatic Zone

**FIG. 10.** Equivalent Air Temperature \( T_e \)-Time Relationship Characteristic of North Italy’s Climatic Zone for Different Parts of Section

**FIG. 11.** Finite-Element Mesh of Box-Girder Analysis

**FIG. 12.** Internal Temperature, Equivalent Air Temperature \( ^\circ{C} \), and Principal Stresses (MPa) Profiles at Midnight (Initial Conditions of Third Day of Analysis)

**FIG. 13.** Internal Temperature, Equivalent Air Temperature \( ^\circ{C} \), and Principal Stresses (MPa) Profiles at 6:00 a.m. of Third Day of Analysis
**Numerical Results**

The stress and strain fields are evaluated within the hypothesis of a two-dimensional plain-strain problem. The assumption reproduces the conditions of any transversal section of a theoretical infinitely long bridge. Such conditions also involve a two-dimensional heat flux.

Fig. 11 shows the finite-element mesh used in the analysis. The choice of such simple discretization is because this analysis is only a validation example, not the real modeling of an existing structure, for which a much more complex mesh should be necessary.

The symmetry of the problem and the boundary conditions allow the consideration of only half the bridge section in the numerical analysis.

The numerical simulation covered a 3-day period. During the first 48 hr, only the thermal transient is solved, and the actual temperature distribution reached at the end of the period is kept as the initial condition for successive analysis. The July $T^*$ curves in Fig. 9 are employed.

During the last 24 hr of simulation, both the thermal and mechanical problems are solved, and the results of the principal stresses and the temperature profiles at 0:00 a.m. (initial condition), 6:00 a.m., 10:00 a.m., and 3:00 p.m. of the third day are shown in Figs. 12–15.

Comparisons between the external and internal temperature evolution underline the delay, due to the thermal inertia, with which the internal temperatures conform to the external condition. The temperature trend shows no appreciable oscillation in the center of the thicker zone, while more sudden changes take place in the superficial thickness. As a result, tensile stresses originate in the zones at a lower temperature, and compression stresses originate at a higher temperature.

The evaluated stresses have a maximum in the lateral wings of the section at 3:00 p.m., when the upper surface reaches the maximum temperature. The value of tensile force in those zones cannot be neglected and may generate considerable cracking effects, if the reinforced-concrete section is not carefully designed.
Temperature profiles along the symmetric axis of the cross section, numerically evaluated at different time steps, are compared with the same experimentally measured quantities in a similar bridge section (Hoffman et al. 1980) operating under the same exposure conditions (Fig. 16). The good agreement with the experimental data confirms the effectiveness and reliability of the proposed method.

A more accurate analysis might be carried out by considering both the creep effect and nonlinear behavior of the material.

CONCLUSIONS

As shown by the examples, i.e., the stress analyses in a RCC gravity dam and in a box-girder section, the proposed numerical procedure was shown to be effective and of practical applicability in predicting the stress level induced by thermal transients in concrete structures.

To neglect the variation of the thermal characteristics with the stress-strain state of the material allows the study of the problem in a simplified uncoupled way: once the thermal transient is solved, the mechanical problem can be found through the constitutive law of the material. These basic hypotheses make the thermal transient a dynamic problem (i.e., it requires integration along time dimension), and the stress-strain analysis becomes a static problem.

The proposed two-dimensional solution (plain stress and plain strain or axially symmetric problems), and the three-dimensional one, gives the self-balanced stresses into the structure. The averaged temperature and its mean gradient may be easily obtained by the numerical integration of the temperature profile on the section's area. These values may be suitably used to detect the hyperstatic reaction of the external restraints and internal actions due to the structure’s global deformation.

Moreover, the possibility to consider environmental conditions variable in time, internal heat generation, the presence of different concrete mixtures in the same structure, and the variability of the geometry during the analysis allows the investigation of the structure’s construction step-by-step, by simulating successive phases of building and by calculating the related thermal-stress-strain states. This method represents a new approach to the structural design of concrete structure subjected to variable thermal load, and with internal heat generation, and it can be considered an initial attempt to solve the complex problem of transient thermal analysis of monolithic concrete structures. Further developments are needed to include: (1) Nonlinearity of the phenomenon (interaction between temperature and stress-strain fields); (2) material nonlinearity, shrinkage, and cracking behavior of concrete; and (3) time-dependent effects (in particular creep).

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APPENDIX I. REFERENCES


APPENDIX II. NOTATION

The following symbols are used in this paper:

- $a$ = absorption coefficient of surface;
- $C_n$ = matrix of thermal capacity of system;
- $C_p$ = specific heat capacity of concrete;
- $D_{el}$ = tensor of elastic constant;
- $d$ = diffuse solar radiation power reaching surface;
- $E$ = Young modulus;
- $f_s$ = known function on boundary;
- $H$ = time rate of heat generated per unit volume;
- $H(t)$ = known function of velocity of heat generation into unit volume;
- $i$ = direct solar radiation power reaching surface;
- $K_s$ = matrix of heat conductivity of system;
- $N_{,n} = \partial N/\partial x_n = \text{derivative of shape functions};$
- $N_i$ = shape function;
- $n$ = unit vector normal to surface;
- $Q_i$ = vector of nodal heat flux evaluated at time $t$;
- $Q^e_i$ = equivalent convective flux at time $t$;
- $q_i$ = convective heat exchange;
- $q_{ax}$ = thermal power produced by atmosphere and absorbed through material surface;
- $q_e$ = emissivity of surface;
- $q_{eb}$ = thermal power produced by earth and absorbed through material surface;
- $q_{ir}$ = thermal power due to direct and diffuse solar radiation;
- $T$ = temperature;
- $T_a$ = temperature of air;
- $T_e$ = equivalent air temperature;
- $T_i(j, t)$ = temperature value at node $j$ and time $t$;
- $T_i$ = temporal derivative of temperature vector;
- $t_i$ = external load applied on surface $\Sigma_i$;
- $u_i$ = imposed displacements on surface $\Sigma_i$;
- $U$ = displacement field;
- $V$ = external fluid velocity;
- $\alpha$ = convective coefficient;
- $B$ = coefficient of thermal expansion;
- $\Delta T$ = temperature variation;
- $\Delta t$ = time step;
- $\delta_0$ = Kronecker delta;
- $\varepsilon_{ij}$ = total strain tensor;
- $\varepsilon_{0,ij}$ = initial strain;
- $\varepsilon_{r,ij}$ = strain due to thermal expansion;
- $\lambda$ = isotropic thermal conductivity of concrete;
- $\lambda_{nn}$ = matrix of material conductivity coefficients along coordinate direction;
- $\nu$ = Poisson’s ratio;
- $\rho$ = density of material;
- $\Sigma$ = boundary of investigated domain;
- $\Sigma_n$ = bound portion of boundary;
- $\Sigma_f$ = free portion of boundary;
- $\sigma$ = Stefan-Boltzmann constant;
- $\sigma_0$ = Cauchy stress tensor; and
- $\sigma_{n,ij}$ = initial stress corresponding to $\varepsilon_{0,ij}$. 

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